

# Machine Vision

## Chapter 2: Image Preprocessing

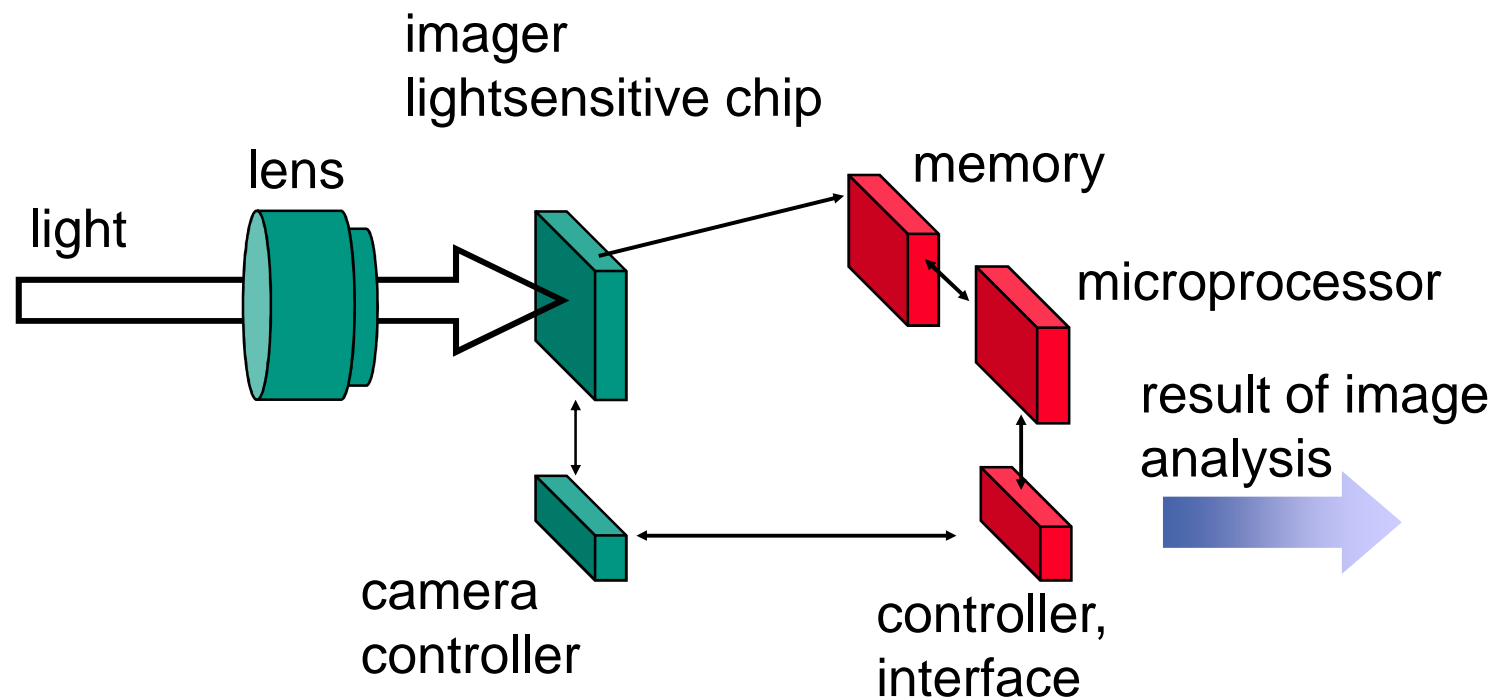
*Dr. Martin Lauer* Institut für Mess-  
und Regelungstechnik



# Image Formation and Analysis

electronic camera  
(formation)

ECU (electronic control unit)  
(image processing)



# Imager

- Process of image formation:

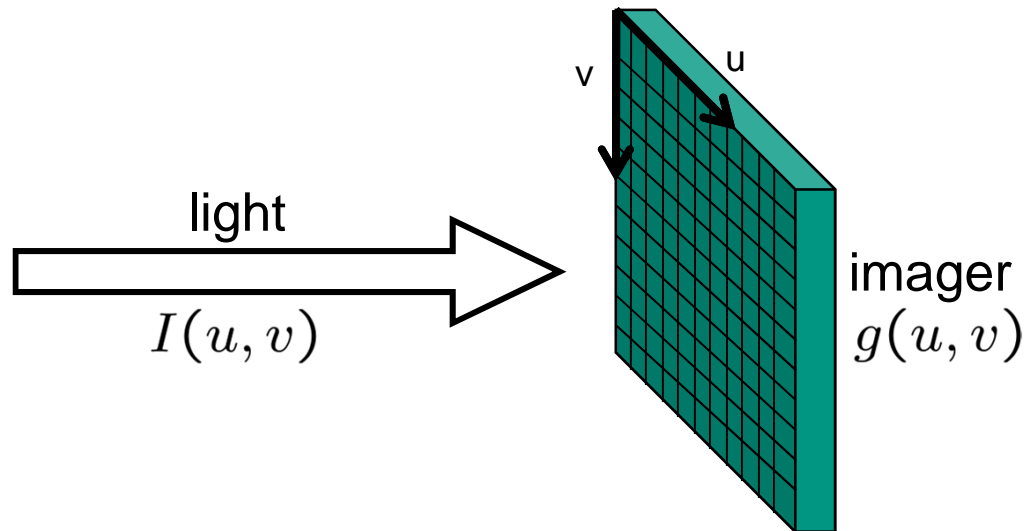
- incident light intensity:

$$I : \mathbb{R}^2 \rightarrow \mathbb{R}$$

- output of imager:

$$g : \{0, \dots, w - 1\} \times \{0, \dots, h - 1\} \rightarrow \{0, \dots, g_{max}\}$$

$w, h$  : image width, height



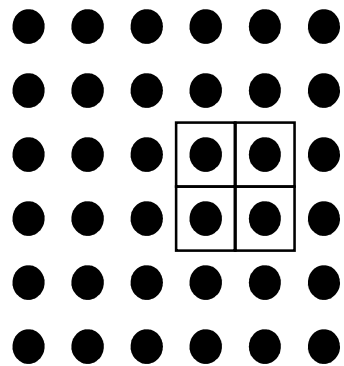
# Imager

- Process of image formation:
  - **sampling**  
evaluate light intensity on a regular grid of points
  - **quantization**  
map continuous signals to discrete values (natural numbers)
  - **blur and noise**
  - **color**  
will be discussed later. Here: only light intensity/grey level images

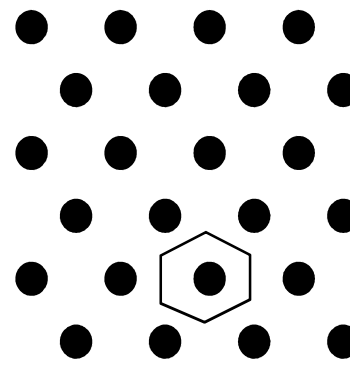


# Sampling

- 2D grids used for sampling



rectangular



hexagonal

- electronic cameras: rectangular, equidistant grids
- biology: hexagonal grids with varying resolution

# Sampling: Moiré Patterns

- Moiré patterns
  - sampling might cause artifacts

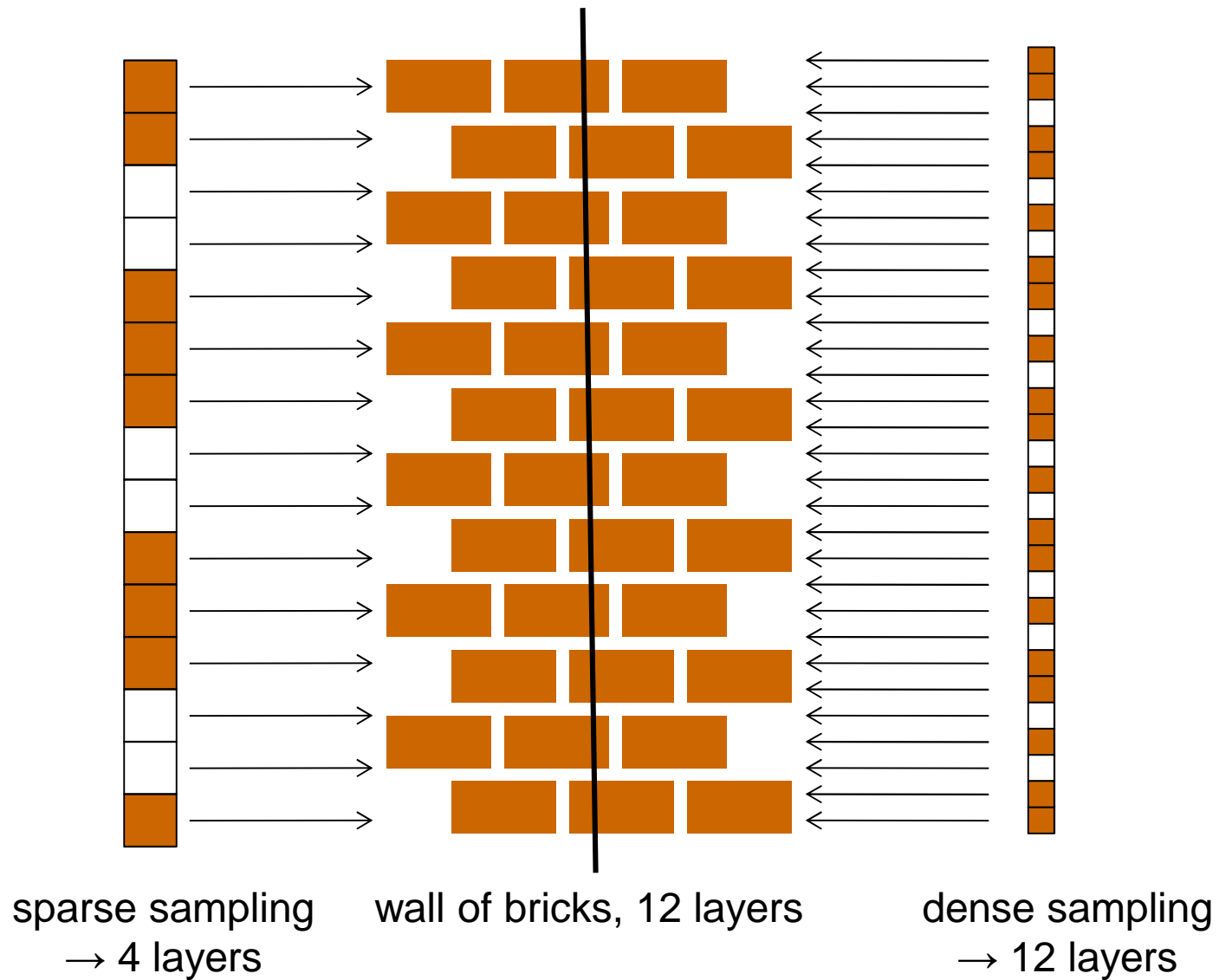


original picture



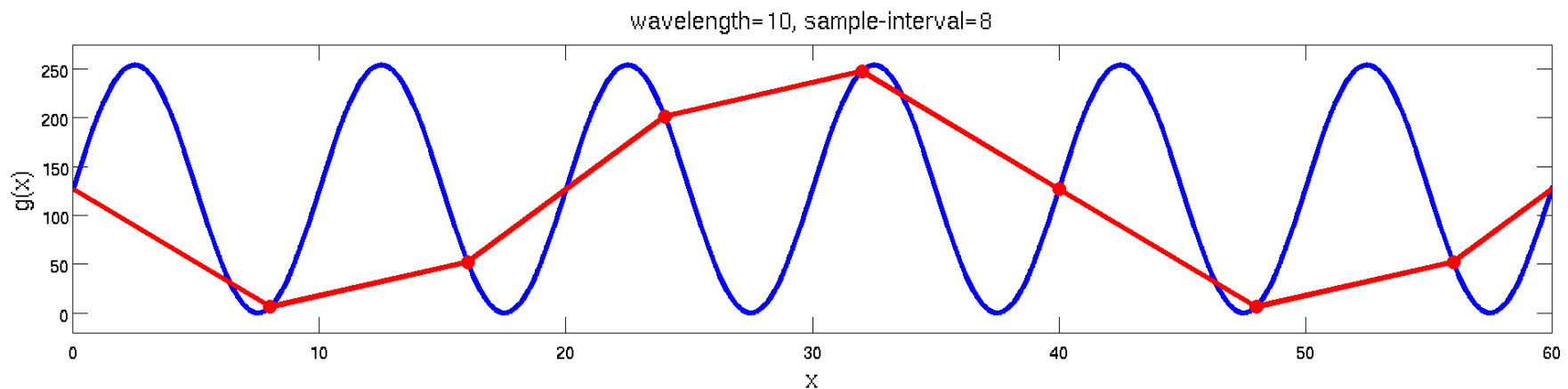
picture with Moiré pattern

# Sampling: Moiré Patterns



## Sampling: Moiré Patterns cont.

- 1D-example of Moiré patterns:



The occurrence of Moiré patterns depends on the sampling rate compared to the maximal frequency of the signal (image)

# Nyquist-Shannon Sampling Theorem

If  $f$  is band bounded signal with cutoff frequency  $k_0$  then it is completely determined by giving its ordinates at a series of points spaced at most  $\frac{1}{2k_0}$ , i.e. the sample frequency must be larger than  $2k_0$

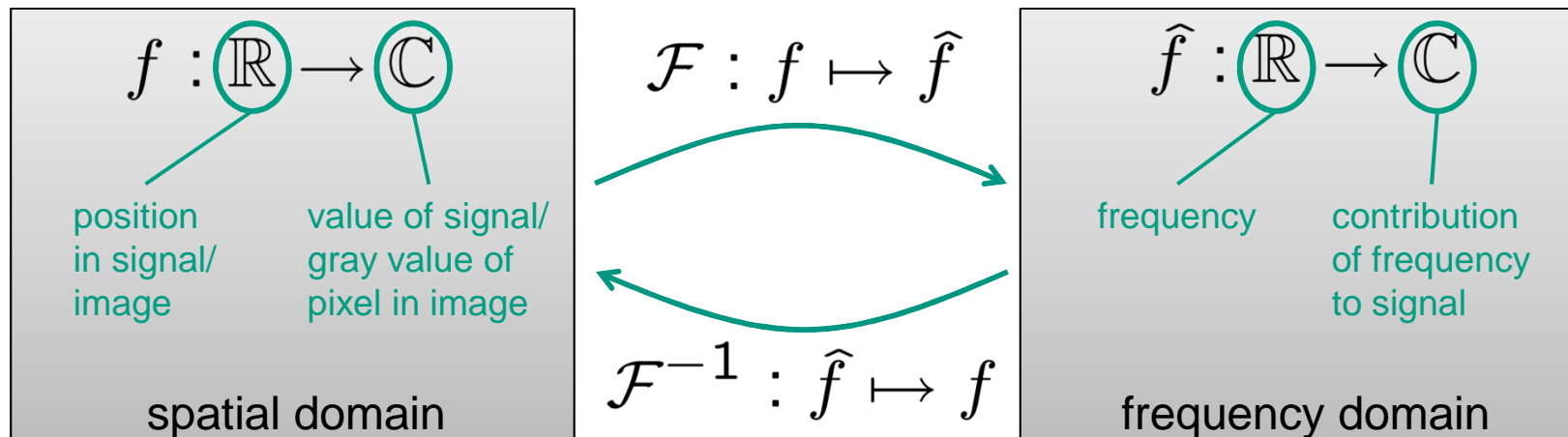
- Questions:
  - what is a band-bounded signal?
  - what is a cutoff frequency?

# Fourier Transform

- Assume a periodic signal

$$f : \mathbb{R} \rightarrow \mathbb{C}$$

- Then, we can define the **Fourier transform** of  $f$



$$\hat{f}(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i k x} dx$$

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(k) e^{2\pi i k x} dk$$

# Fourier Transform

- Properties:

- the Fourier transform is **linear**

$$\mathcal{F}\{\alpha f(x) + \beta g(x)\}(k) = \alpha \hat{f}(k) + \beta \hat{g}(k)$$

- **shifting** a signal along the x-axis only changes the complex angles in frequency domain but not the amplitudes

$$\mathcal{F}\{f(x - \xi)\}(k) = e^{-2\pi i \xi k} \hat{f}(k)$$

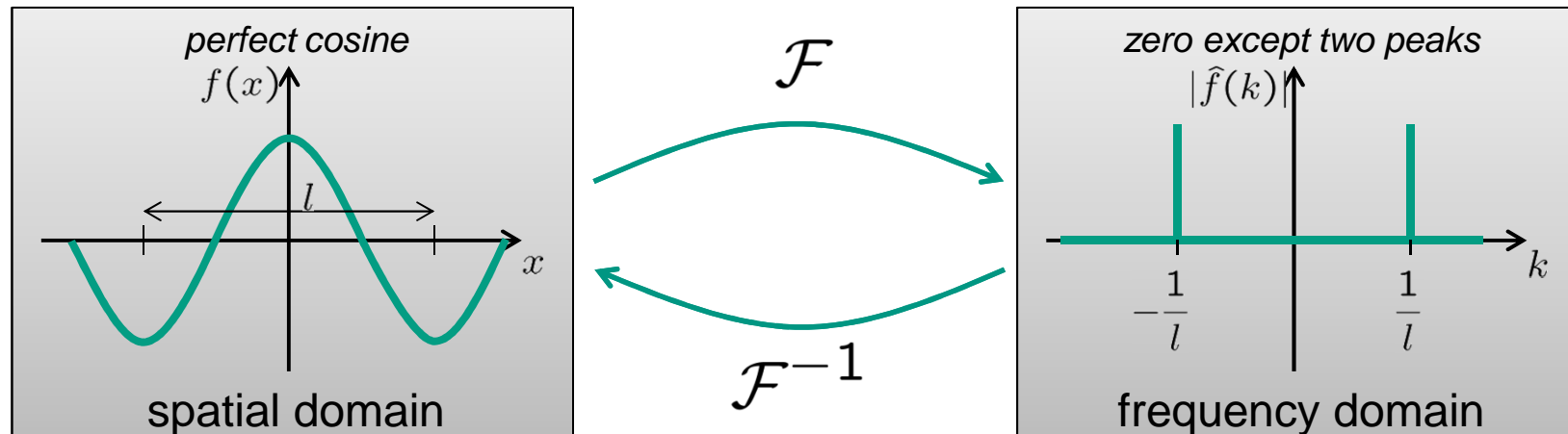
- **rescaling** the x-axis in the spatial domain rescales the frequency axis in a reciprocal way

$$\mathcal{F}\{f(\alpha x)\}(k) = \frac{1}{|\alpha|} \hat{f}\left(\frac{k}{\alpha}\right)$$

# Fourier Transform

- Properties:

- a cosine in spatial domain generates two peaks in frequency domain



- the peaks are located at position reciprocal to the period length
- if the signal in spatial domain is a linear combination of cosines, the Fourier transform will be a set of peaks in frequency domain
- intuitive interpretation: the Fourier transform decomposes a periodic signal into a (potentially infinite) linear combination of cosines



# Fourier Transform

- Observation

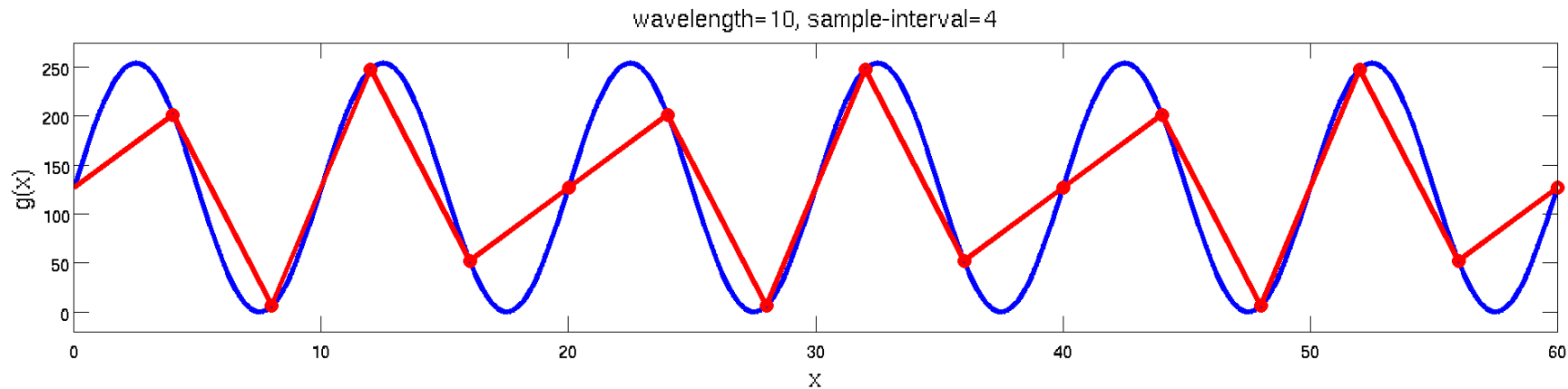
- smooth periodic functions with small slope can be composed out of cosines with large period
- periodic functions with large slope require cosines with small period
- periodic functions that are discontinuous or have discontinuous derivatives require cosines with unbounded frequencies

- Definition

A signal  $f$  is band bounded with cutoff frequency  $k_0$  if its Fourier transform is zero for all frequencies larger than the cutoff frequency, i.e.

$$\hat{f}(k) = 0 \text{ for all } k \text{ with } |k| \geq k_0$$

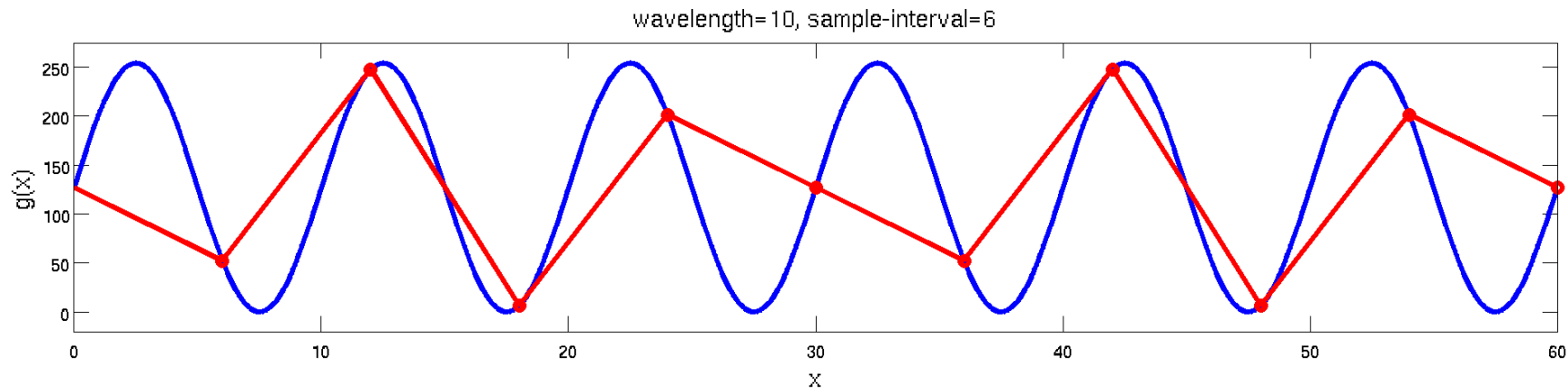
# Nyquist-Shannon Sampling Theorem



- signal is band bounded (sine function)
- sampling frequency high enough
$$f_{sample} = \frac{1}{4} > 2f_{signal} = \frac{2}{10}$$
- reconstruction of the signal possible

If  $f$  is band bounded signal with cutoff frequency  $k_0$  then it is completely determined by giving its ordinates at a series of points spaced at most  $\frac{1}{2k_0}$ , i.e. the sample frequency must be larger than  $2k_0$

# Nyquist-Shannon Sampling Theorem



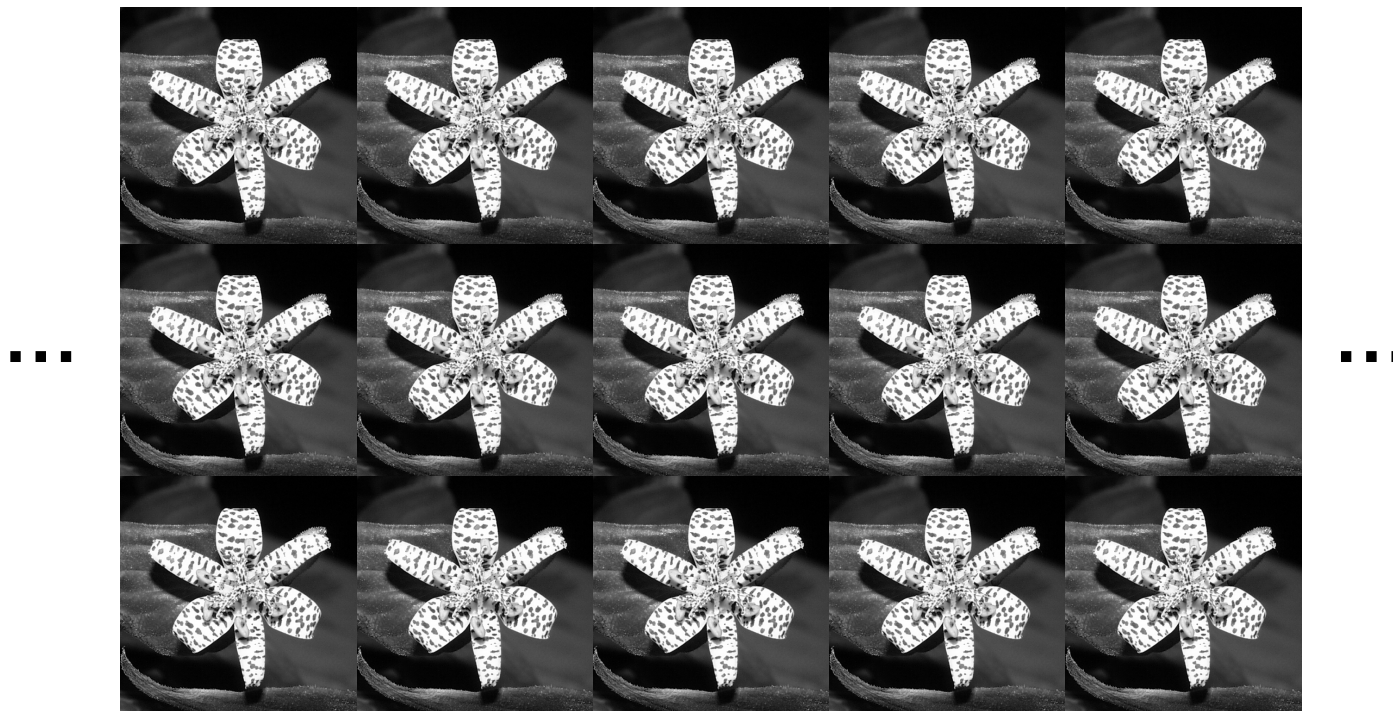
- signal is band bounded (sine function)
- but
$$f_{sample} = \frac{1}{6} < 2f_{signal} = \frac{2}{10}$$
- reconstruction of the signal impossible

If  $f$  is band bounded signal with cutoff frequency  $k_0$  then it is completely determined by giving its ordinates at a series of points spaced at most  $\frac{1}{2k_0}$ , i.e. the sample frequency must be larger than  $2k_0$

# Sampling Theorem and Images

- Remarks:
  - analysis analogously possible for 2d signals
  - image is not periodic, but we can make it periodic by copying it repeatedly to the left, right, top, and bottom

■ ■ ■



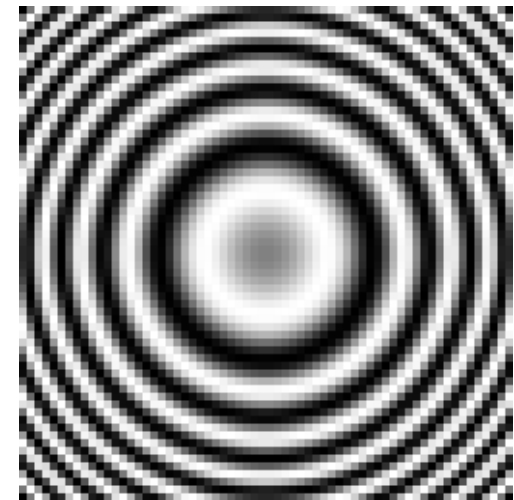
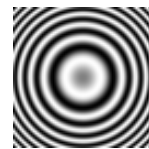
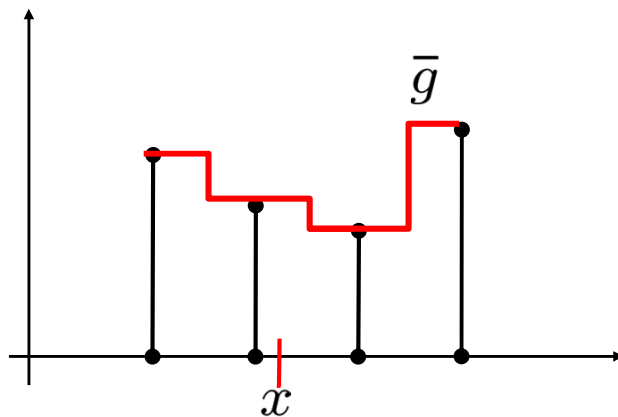
■ ■ ■

# Sampling Theorem and Images

- Questions:
  - how can we determine the sampling frequency of a camera?
  - what can we do if we find that the sampling theorem is violated?

# Image Scaling and Interpolation

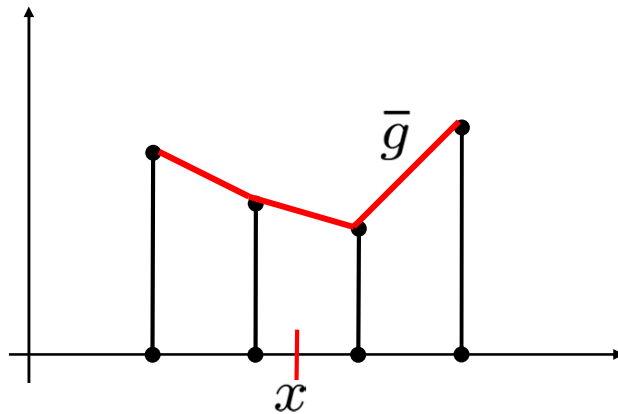
- changing the image size
- scaling needs evaluation of the image at non-integer positions → interpolation
- nearest neighbor interpolation:
  - approximating the grey level function with a step function
  - take the grey value of the nearest integer position
  - problem: aliasing



## Interpolation cont.

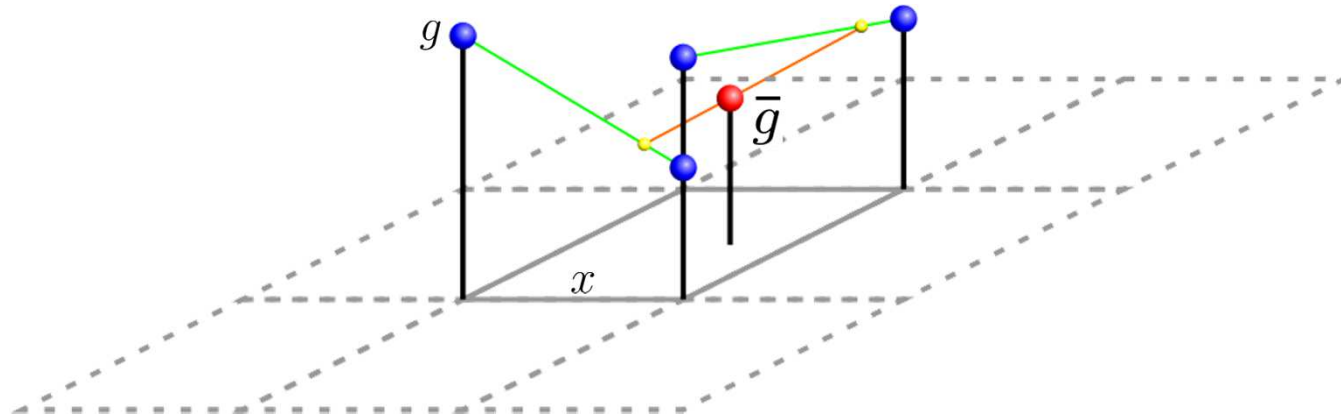
- linear interpolation in ID
  - fit linear function locally around  $x$

$$\bar{g}(x) = g(\lfloor x \rfloor) + (x - \lfloor x \rfloor)(g(\lfloor x \rfloor + 1) - g(\lfloor x \rfloor))$$



## Interpolation cont.

- extension of linear interpolation to 2D:



- interpolate from 4 neighboring pixels



## Interpolation cont.

- cubic interpolation
  - fit cubic polynomial to the grey level

– solve

$$\bar{g}(x) = a \cdot (x - \lfloor x \rfloor)^3 + b \cdot (x - \lfloor x \rfloor)^2 + c \cdot (x - \lfloor x \rfloor) + d$$

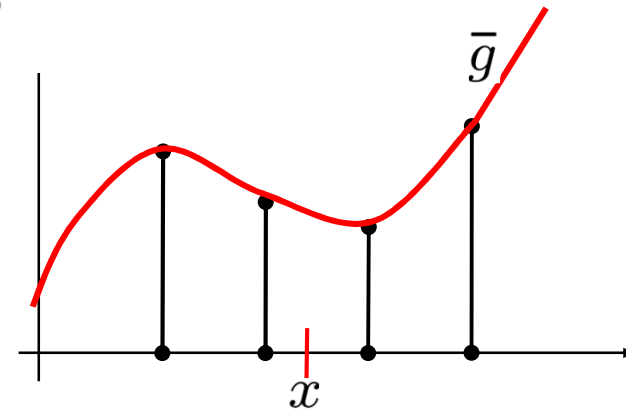
yields:

$$a = -\frac{1}{6}g(\lfloor x \rfloor - 1) + \frac{1}{2}g(\lfloor x \rfloor) - \frac{1}{2}g(\lfloor x \rfloor + 1) + \frac{1}{6}g(\lfloor x \rfloor + 2)$$

$$b = \frac{1}{2}g(\lfloor x \rfloor - 1) - g(\lfloor x \rfloor) + \frac{1}{2}g(\lfloor x \rfloor + 1)$$

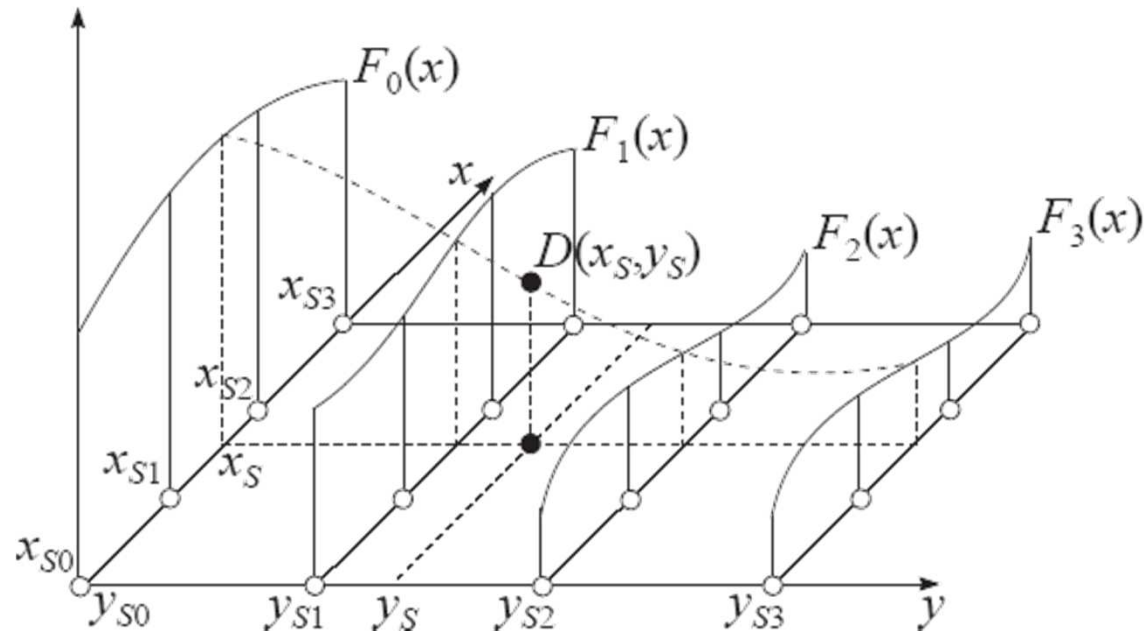
$$c = -\frac{1}{3}g(\lfloor x \rfloor - 1) - \frac{1}{2}g(\lfloor x \rfloor) + g(\lfloor x \rfloor + 1) - \frac{1}{6}g(\lfloor x \rfloor + 2)$$

$$d = g(\lfloor x \rfloor)$$



## Interpolation cont.

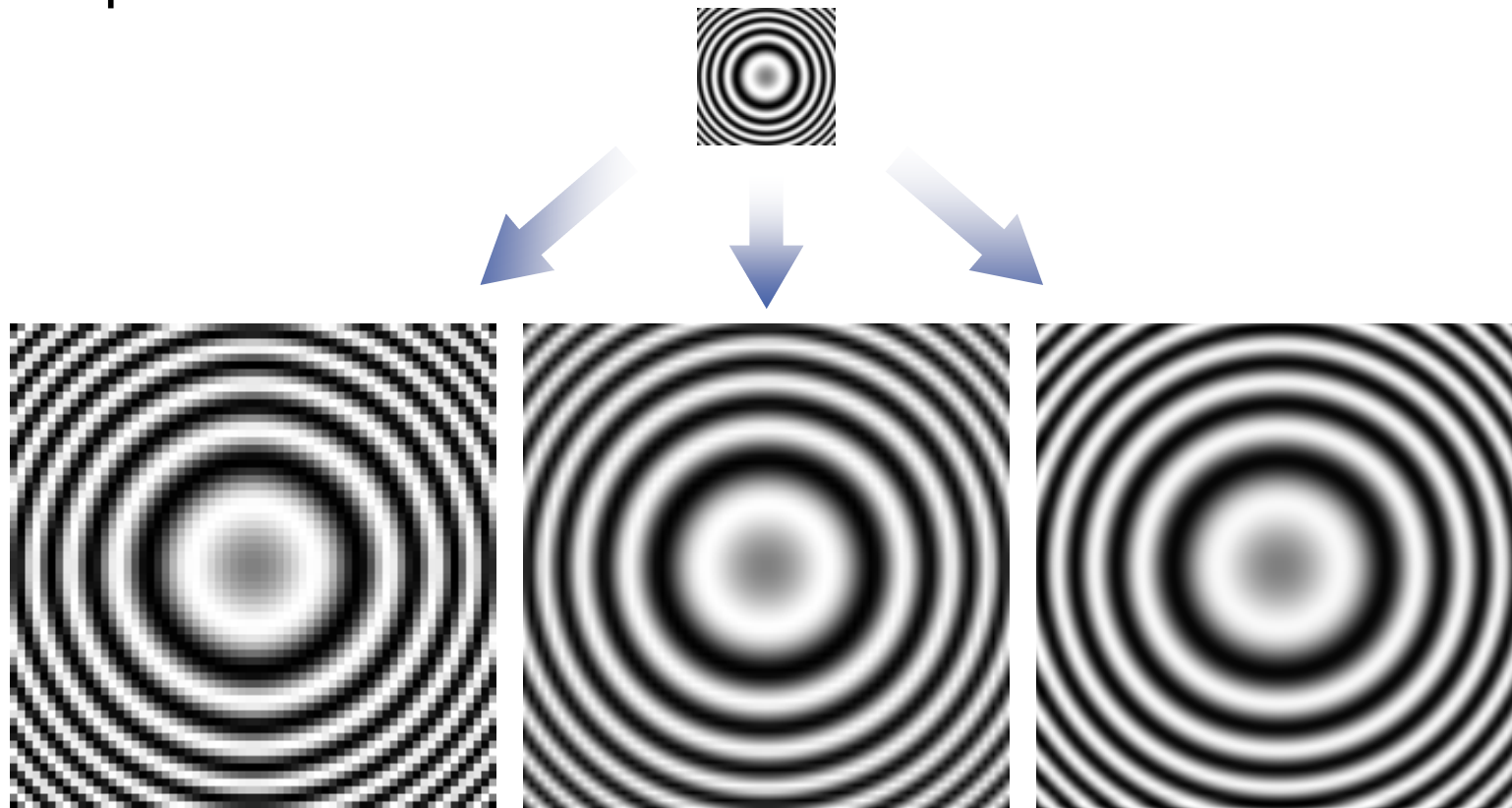
- extension of cubic interpolation to 2D:



- interpolation from 16 neighboring pixels

## Interpolation cont.

- Example:



nearest neighbor

linear interpolation

cubic interpolation

# Imager

- Process of image formation:
  - **sampling**  
evaluate light intensity on a regular grid of points
  - **quantization**  
map continuous signals to discrete values (natural numbers)
  - **blur and noise**
  - **color**  
will be discussed later. Here: only light intensity/grey level images

# Quantization

- incident light:

$$I : \mathbb{R}^2 \rightarrow \mathbb{R}$$

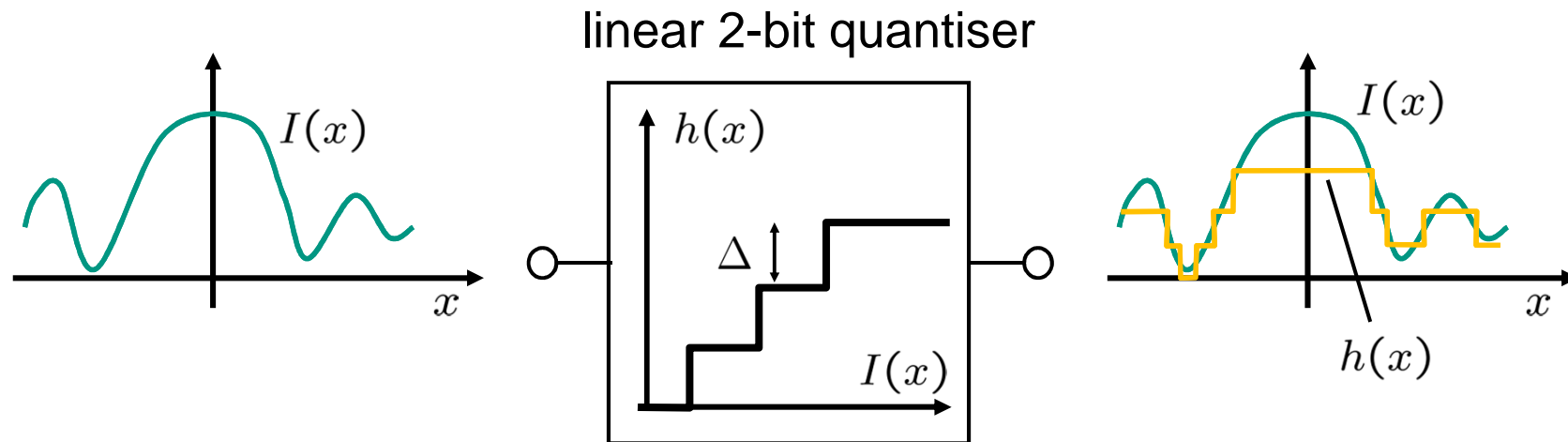

- digital camera signals:

$$g : \{0, \dots, w - 1\} \times \{0, \dots, h - 1\} \rightarrow \{0, \dots, g_{max}\}$$

$w, h$  : image width, height

- need transformation from real valued light intensity to discrete digital signals (analog-to-digital converter)

## Quantization cont.



- characteristic with equidistant steps (“linear”) of size  $\Delta$ :

$$g(x) = \max\{0, \min\{g_{max}, \left\lfloor \frac{I(x)}{\Delta} + \frac{1}{2} \right\rfloor\}\}$$

$$h(x) = \Delta g(x)$$

- error of non-overdriven quantiser:

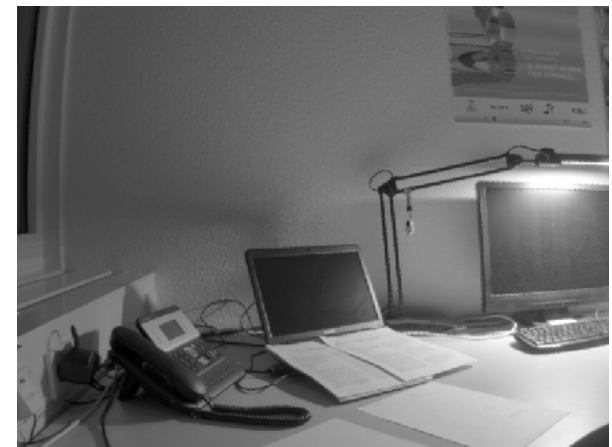
$$I(x) - h(x) \in \left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$$

## Quantization cont.

- characteristic of digital cameras:
  - linear
  - logarithmic
- grey level cutoff value
  - 1 (binary images, “bitmaps”)  $\rightarrow$  1 bit/pixel
  - 255  $\rightarrow$  8 bit/pixel = 1 byte/pixel
  - 4095  $\rightarrow$  12 bit/pixel = 1.5 byte/pixel
  - 65535  $\rightarrow$  16 bit/pixel = 2 byte/pixel
- correction of grey level distribution
  - image too dark/too bright
  - low contrast
  - non-linear camera characteristic

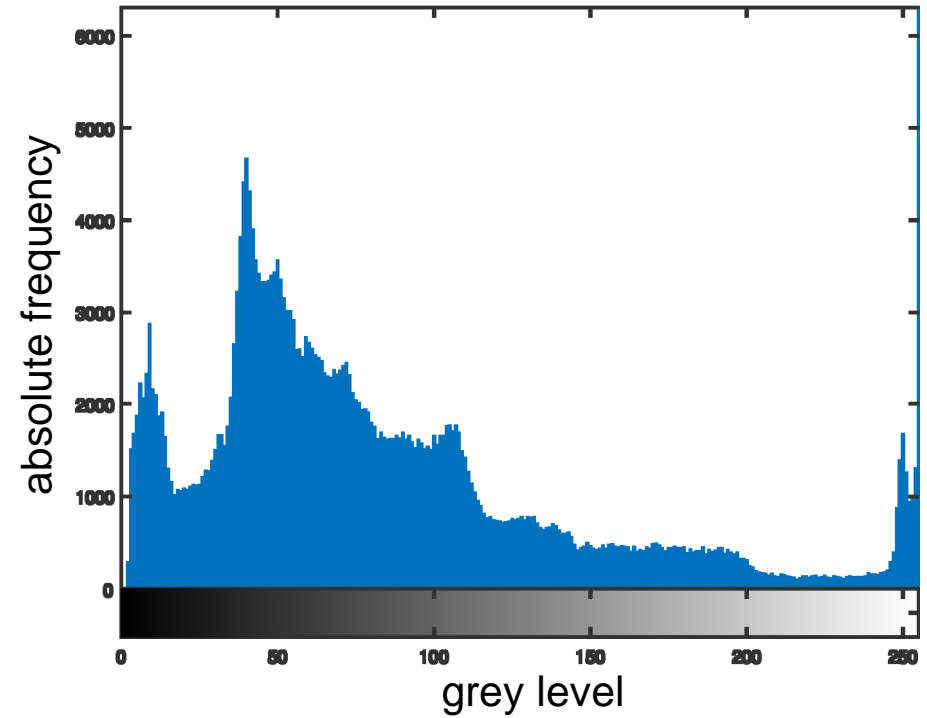


linear characteristic



logarithmic characteristic

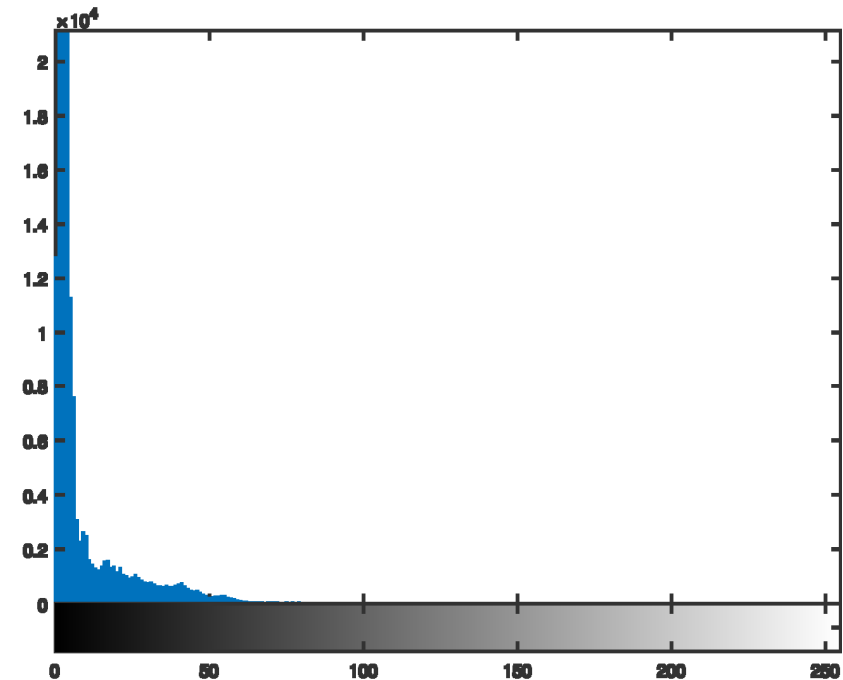
# Grey Level Histogram



- grey level histograms display distribution of grey levels



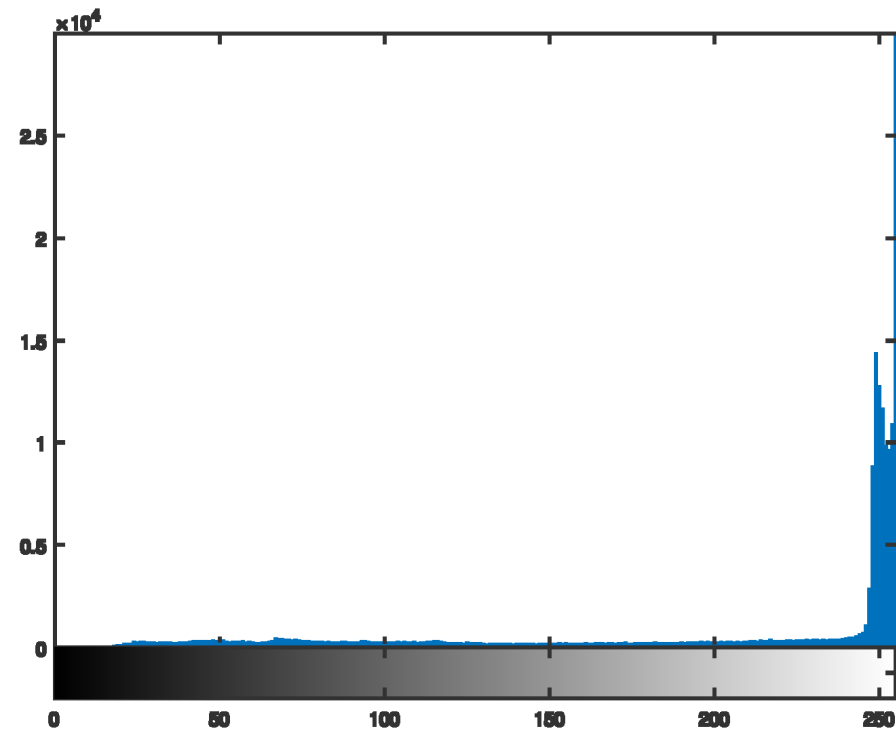
## Grey Level Histogram cont.



### Underexposed images:

- open aperture of camera
  - increase exposure time of camera
  - increase gain
  - add additional light sources
- multiply grey values by a constant
  - auto-exposure implemented in many digital cameras

## Grey Level Histogram cont.



### Overexposed images:

- information loss due to cutoff value, no reconstruction possible
- close aperture of camera
- reduce exposure time of camera
- auto-exposure

## Grey Level Histogram cont.



$\gamma = 1$



$\gamma = 0.5$

$\gamma = 2$

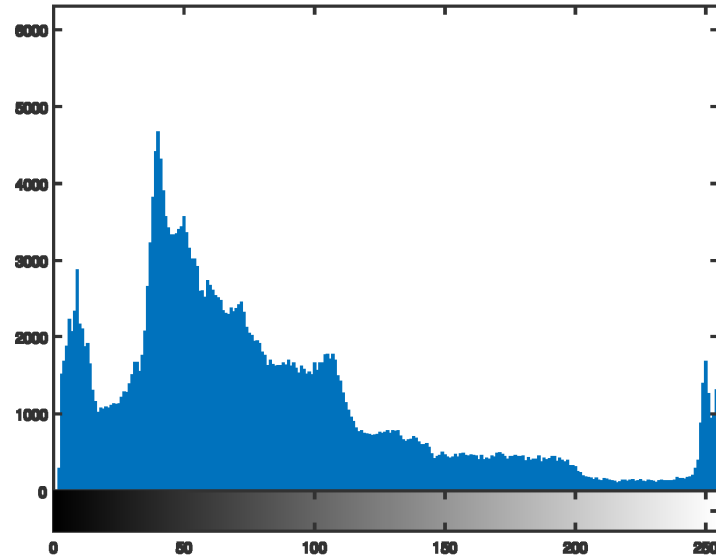


### Gamma correction:

$$g_{out} = g_{max} \left( \frac{g_{in}}{g_{max}} \right)^{\gamma}$$

- keeps black and white
- nonlinear transformation

# Grey Level Histogram cont.



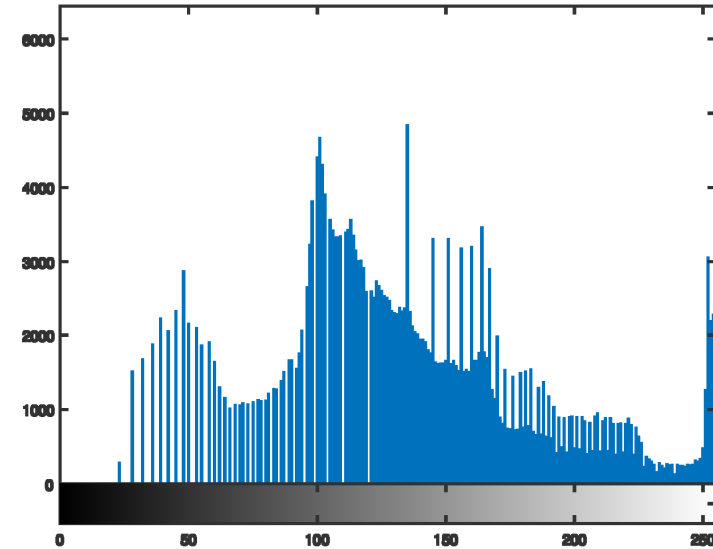
$\gamma = 1$

## Gamma correction:

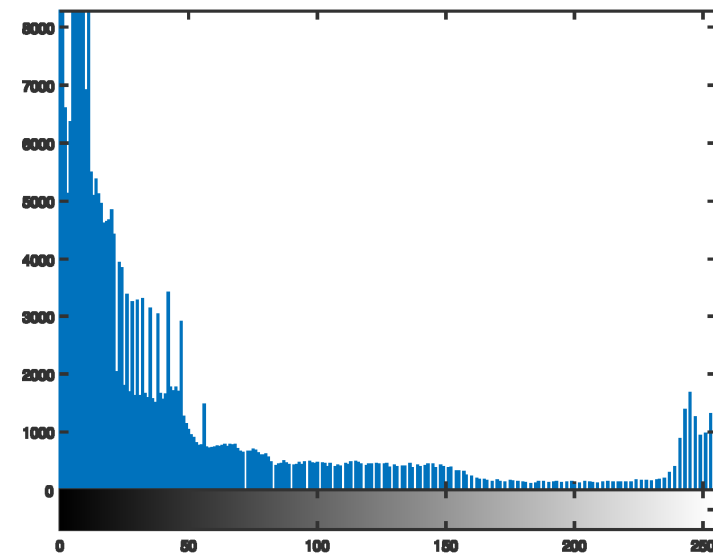
$$g_{out} = g_{max} \left( \frac{g_{in}}{g_{max}} \right)^\gamma$$

- keeps black and white
- nonlinear transformation

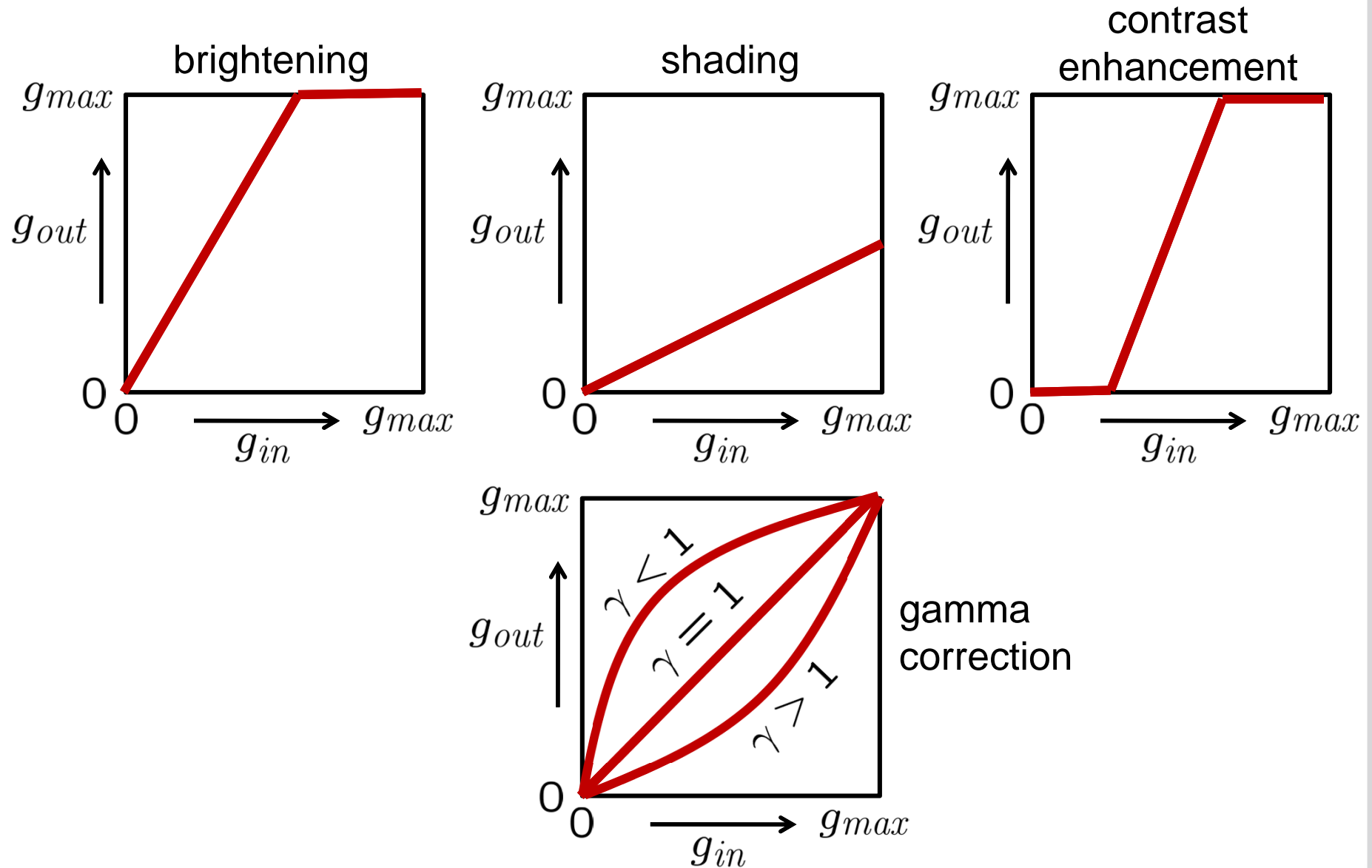
$\gamma = 0.5$



$\gamma = 2$

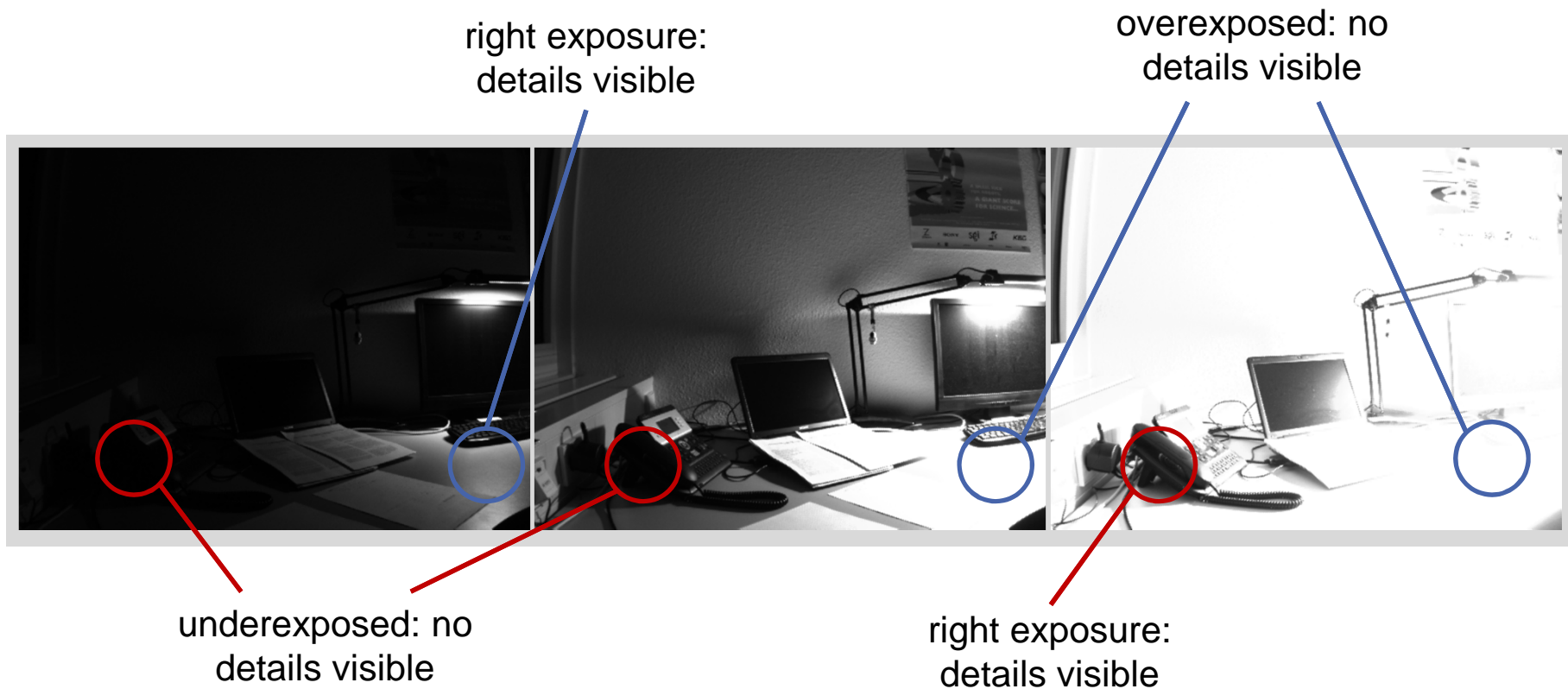


# Grey Level Transformations

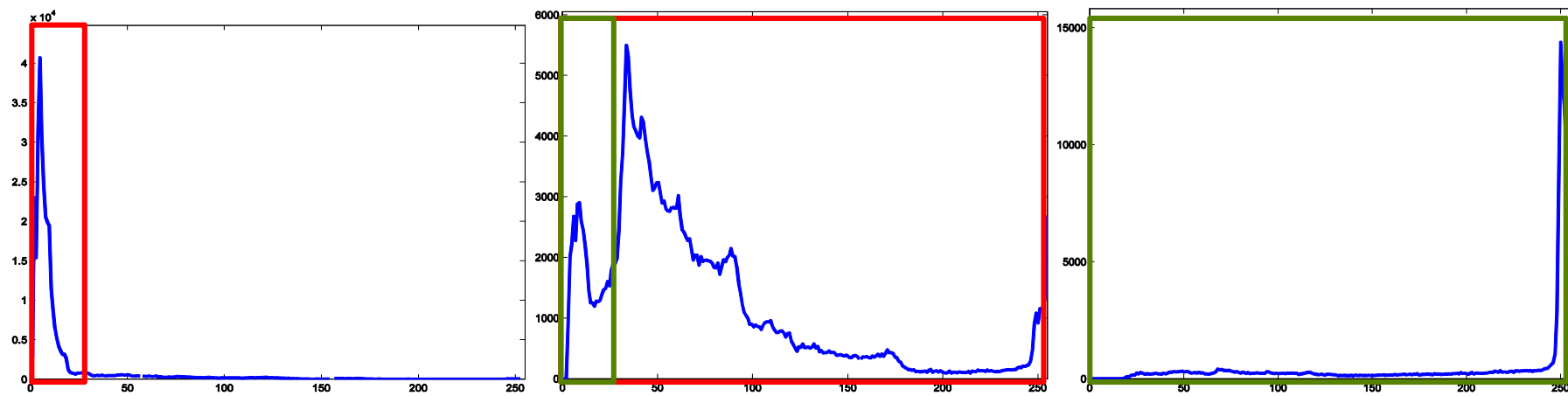


# Exposure Series

- exposure bracketing, high dynamic range imaging (HDRI):  
increase the grey value resolution combining over- and underexposed images



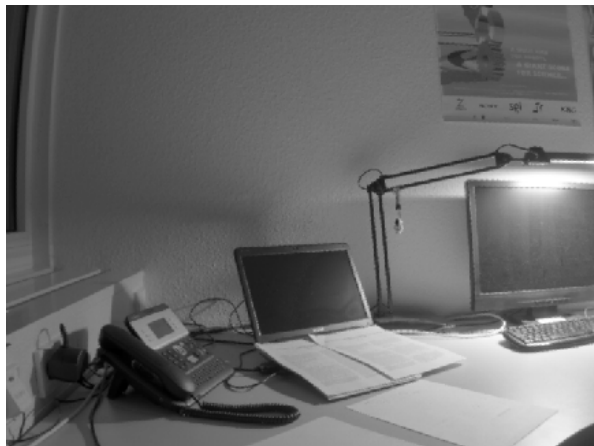
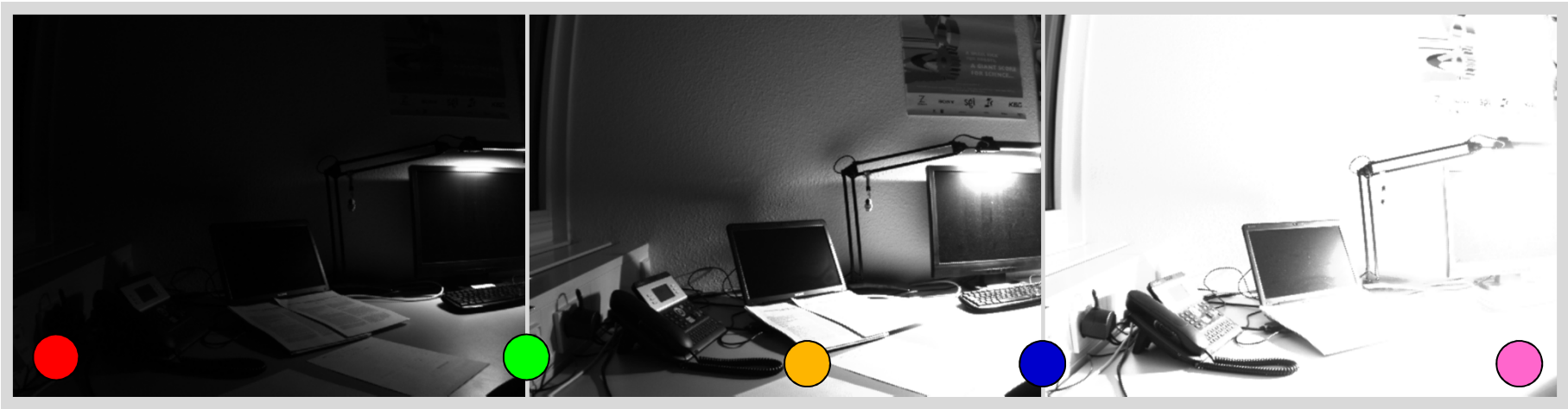
## Exposure Series cont.



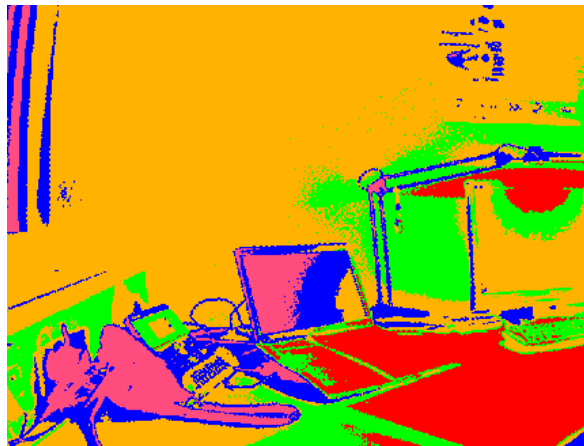
gray level histograms

gray scales differ by a constant factor

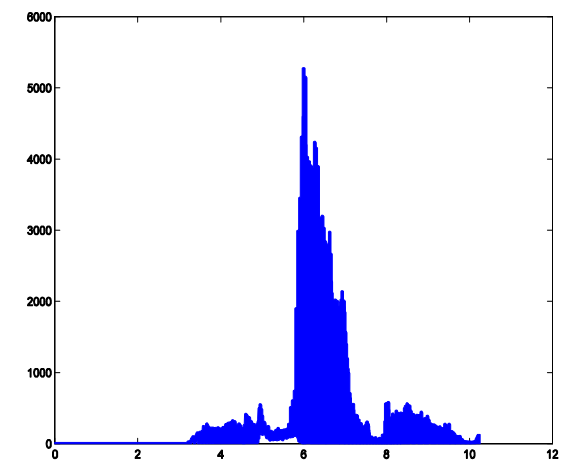
## Exposure Series cont.



HDRI image (after log transform)



HDRI mixing



HDRI histogram  
(after log transform)



# Imager

- Process of image formation:
  - **sampling**  
evaluate light intensity on a regular grid of points
  - **quantization**  
map continuous signals to discrete values (natural numbers)
  - **blur and noise**
  - **color**  
will be discussed later. Here: only light intensity/grey level images

# Convolution Operator

The **convolution** operator

- takes two functions  $f, g$
- creates a new function  $h = g * f$
- which is defined pointwise by

$$h(x) = \int_{-\infty}^{\infty} f(\tau)g(x - \tau)d\tau$$

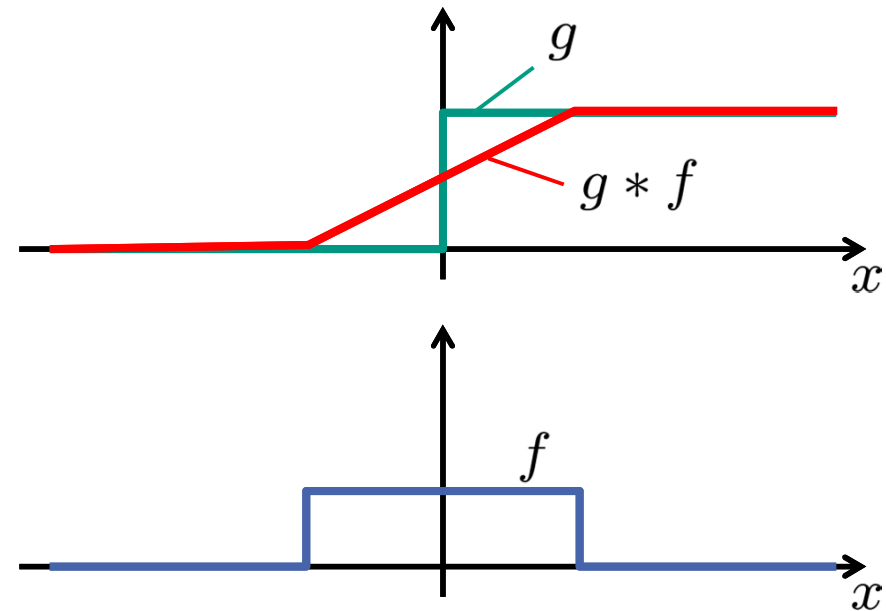
- we interpret
  - $g$  is a gray level image
  - $f$  is a filter function
  - $h$  is a filtered image
- convolution implements a linear filter

# Convolution Operator

- Example

$$g(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

$$f(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{1}{2} & \text{if } -1 \leq x \leq +1 \\ 0 & \text{if } x > +1 \end{cases}$$



$$\begin{aligned} (g * f)(x) &= \int_{-\infty}^{\infty} f(\tau)g(x - \tau)d\tau = \int_{-\infty}^x f(\tau) \cdot 1d\tau + \int_x^{\infty} f(\tau) \cdot 0d\tau \\ &= \int_{-\infty}^x f(\tau)d\tau = \begin{cases} 0 & \text{if } x < -1 \\ \frac{1}{2}(x + 1) & \text{if } -1 \leq x \leq +1 \\ 1 & \text{if } x > +1 \end{cases} \end{aligned}$$

# Convolution Operator

- Properties of convolution

- commutativity

$$f * g = g * f$$

- associativity

$$(f * g) * h = f * (g * h)$$

- linearity

$$f * (\alpha g + \beta h) = \alpha(f * g) + \beta(f * h)$$

- relationship with Fourier transform

$$\mathcal{F}(f * g) = \mathcal{F}(f) \cdot \mathcal{F}(g)$$

$$\mathcal{F}(f \cdot g) = \mathcal{F}(f) * \mathcal{F}(g)$$

# Convolution of Images

- Convolution can be extended

– to the 2d case

$$f, g : \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$(g * f)(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tau, \rho) g(x - \tau, y - \rho) d\tau d\rho$$

– to the case of function which we can evaluate only at integer positions

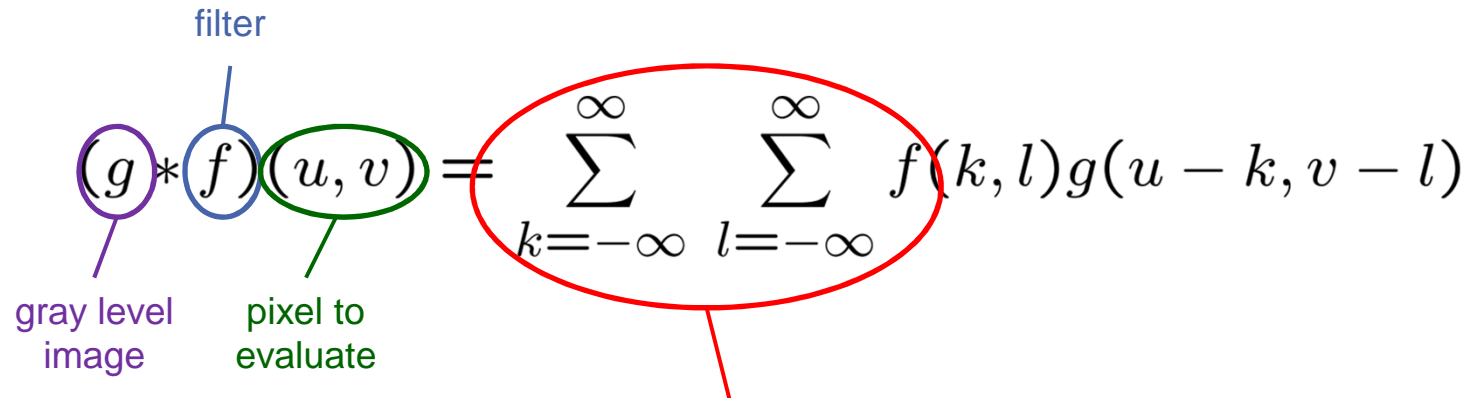
$$f, g : \mathbb{Z} \rightarrow \mathbb{R}$$

$$(g * f)(u) = \sum_{k=-\infty}^{\infty} f(k) g(u - k)$$

$$f, g : \mathbb{Z}^2 \rightarrow \mathbb{R}$$

$$(g * f)(u, v) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f(k, l) g(u - k, v - l)$$

# Convolution of Images



The diagram shows the convolution equation  $(g * f)(u, v) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f(k, l)g(u - k, v - l)$ . Annotations include: a purple circle around  $g$  labeled 'gray level image', a blue circle around  $f$  labeled 'filter', and a green circle around  $(u, v)$  labeled 'pixel to evaluate'. A red oval encircles the double summation, with a red line pointing to the text 'problem: sums run to infinity!'.

$$(g * f)(u, v) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f(k, l)g(u - k, v - l)$$

gray level image

filter

pixel to evaluate

problem: sums run to infinity!

– in practice, filters and images have limited size.

We assume that all gray levels outside of filter size are 0

# Convolution of Images

- Example

 $g :$ 

3	4	2	2	2	2	4
5	2	1	1	1	0	2
6	3	2	2	0	1	3
4	4	8	5	4	7	3
6	6	7	8	9	9	7

 $f :$ 

2	2	1
1	0	0
0	1	3

 $g * f :$ 

	30	26	15	12	18	
	47	39	30	32	30	
	61	52	53	57	45	

$$(g * f)(5, 1) =$$

$+ f(-1, -1)g(6, 2)$	$+ f(0, -1)g(5, 2)$	$+ f(1, -1)g(4, 2)$
$+ f(-1, 0)g(6, 1)$	$+ f(0, 0)g(5, 1)$	$+ f(1, 0)g(4, 1)$
$+ f(-1, 1)g(6, 0)$	$+ f(0, 1)g(5, 0)$	$+ f(1, 1)g(4, 0)$

boundary pixels are typically left free  
since convolution requires evaluation  
of pixels outside of image  $g$

$$(g * f)(u, v) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f(k, l)g(u - k, v - l)$$

# Blur and Noise

- types of blur and noise:
  - motion blur
  - defocus aberration
  - statistical noise of sensor cells and amplifiers
  - malfunctioning sensor cells

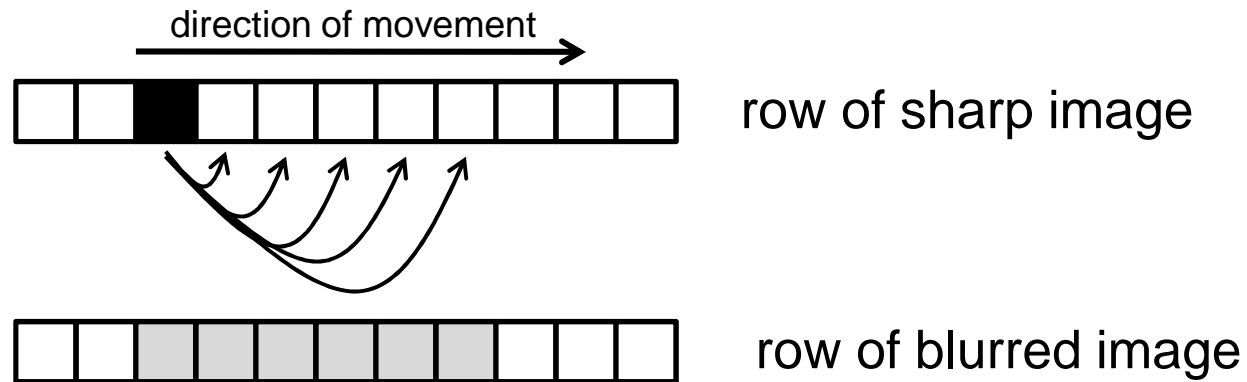


image source: wikipedia

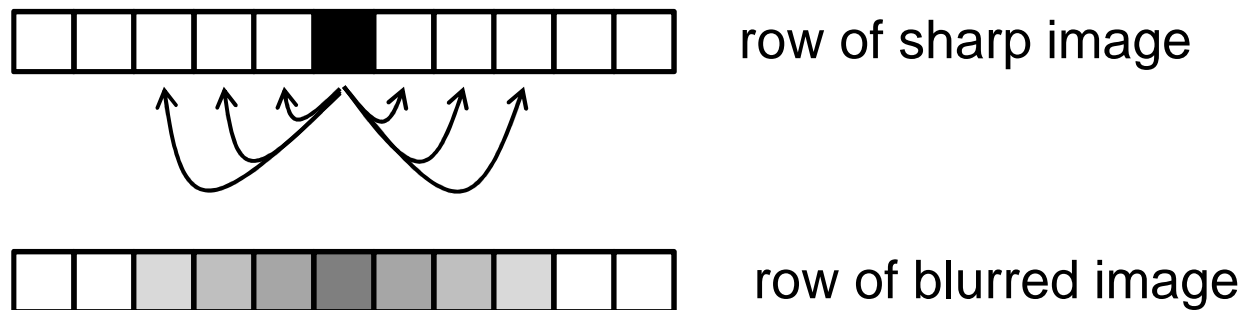


# Models of Blur

- Motion blur:



- Gaussian blur:



# Models of Blur cont.

- blur can be modeled with convolution

$$g_{blurred} = g_{sharp} * p$$

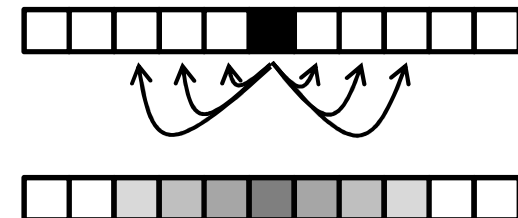
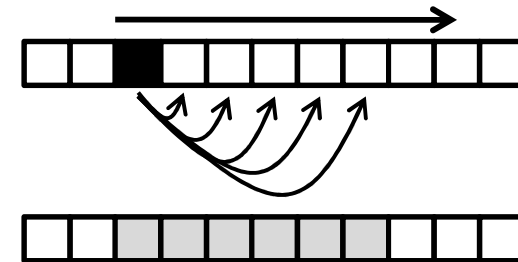
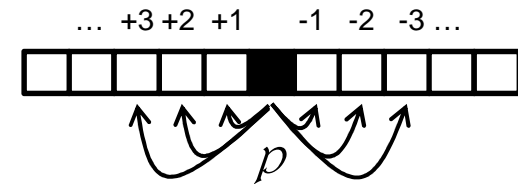
$p$  : “point-spread-function” models blur

- motion blur (along x-axis by  $n$  pixels):

$$p_{motion}(x) = \begin{cases} \frac{1}{n} & \text{if } -n < x \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

- Gaussian blur (with variance  $\sigma^2$ ):

$$p_{Gauss}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$$



# Wiener Deconvolution

- techniques to obtain sharp image from blurred image based on Wiener filter

$$g_{blurred} = g_{sharp} * p + v$$

$p$  : point-spread-function

$v$  : pixel noise

assume  $g_{sharp}$  and  $v$  be independent

$$g_{restored} = f * g_{blurred}$$

find optimal  $f$  that minimizes:

$$e(k) = \mathbb{E} \left[ |\hat{g}_{sharp}(k) - \hat{g}_{restored}(k)|^2 \right]$$

( $\hat{g}$  denotes Fourier transform of  $g$ )

( $\mathbb{E}$  denotes expectation value)

## Wiener Deconvolution cont.

$$e(k) = \mathbb{E} [|\hat{g}_{sharp}(k) - \hat{g}_{restored}(k)|^2]$$

$$= \mathbb{E} [|\hat{g}_{sharp}(k) - \hat{f}(k)\hat{g}_{blurred}(k)|^2]$$

$$= \mathbb{E} [|\hat{g}_{sharp}(k) - \hat{f}(k)(\hat{p}(k)\hat{g}_{sharp}(k) + \hat{v}(k))|^2]$$

$$= \mathbb{E} [|(1 - \hat{f}(k)\hat{p}(k))\hat{g}_{sharp}(k) - \hat{f}(k)\hat{v}(k)|^2]$$

$$= (1 - \hat{f}(k)\hat{p}(k))(1 - \hat{f}(k)\hat{p}(k))^* \mathbb{E} [\hat{g}_{sharp}(k)\hat{g}_{sharp}^*(k)]$$

$$- (1 - \hat{f}(k)\hat{p}(k))\hat{f}^*(k) \mathbb{E} [\hat{g}_{sharp}(k)\hat{v}^*(k)]$$

$$- \hat{f}(k)(1 - \hat{f}(k)\hat{p}(k))^* \mathbb{E} [\hat{v}(k)\hat{g}_{sharp}^*(k)]$$

$$+ \hat{f}(k)\hat{f}^*(k) \mathbb{E} [\hat{v}(k)\hat{v}^*(k)]$$

independence of signal and noise yields:

$$\mathbb{E} [\hat{g}_{sharp}(k)\hat{v}^*(k)] = \mathbb{E} [\hat{v}(k)\hat{g}_{sharp}^*(k)] = 0$$

denote:

$$S(k) = \mathbb{E} [\hat{g}_{sharp}(k)\hat{g}_{sharp}^*(k)], \quad N(k) = \mathbb{E} [\hat{v}(k)\hat{v}^*(k)]$$

$$e(k) = (1 - \hat{f}(k)\hat{p}(k))(1 - \hat{f}(k)\hat{p}(k))^* S(k) + \hat{f}(k)\hat{f}^*(k) N(k)$$

## Wiener Deconvolution cont.

- zeroing the derivative of  $e$  to obtain the minimum yields:

$$\hat{f}(k) = \frac{\hat{p}^*(k)S(k)}{\hat{p}(k)\hat{p}^*(k)S(k) + N(k)} = \frac{\hat{p}^*(k)}{|\hat{p}(k)|^2 + \left(\frac{S(k)}{N(k)}\right)^{-1}}$$

which defines the optimal linear filter (**Wiener filter**)

- $\frac{S(k)}{N(k)}$  is the signal-to-noise ratio

- in the noiseless case:

$$\hat{f}(k) = \frac{1}{\hat{p}(k)} \quad (\text{if } N(k) = 0)$$

- but:  $\frac{S(k)}{N(k)}$  and  $\hat{p}(k)$  must be known

## Wiener Deconvolution cont.



original image

motion blur



restored image



## Wiener Deconvolution cont.



original image

Gaussian blur



restored image



# Models of Noise

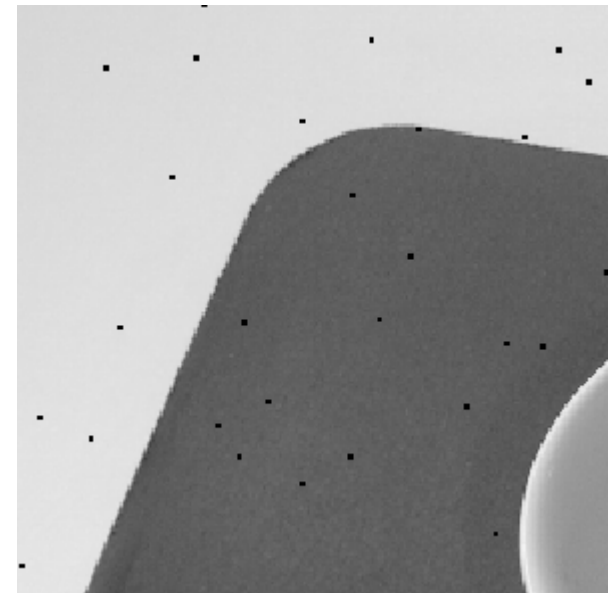
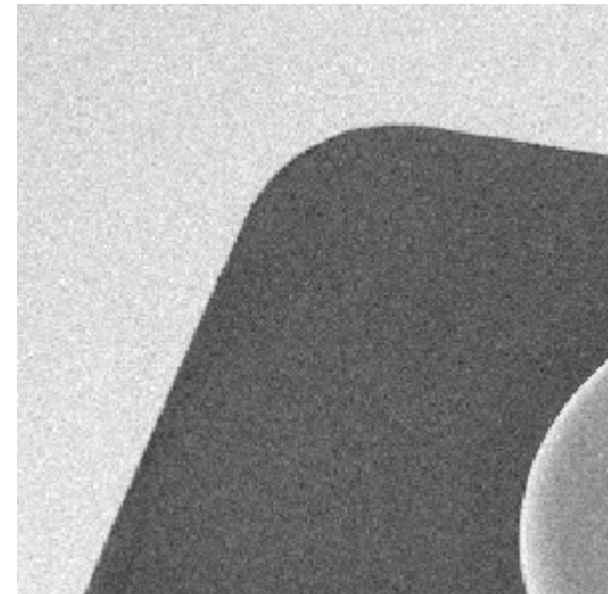
- statistical noise:

$$g_{noisy}(x, y) = g_{sharp}(x, y) + v(x, y)$$
$$v(x, y) \sim N(0, \sigma^2) \text{ i.i.d.}$$

(i.i.d. = independent and identically distributed)

- malfunctioning sensors:

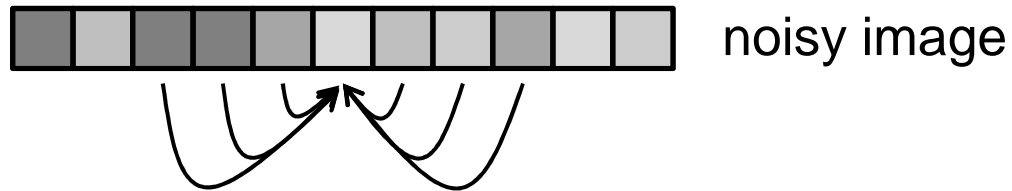
$$g_{noisy}(x, y) = \begin{cases} g_{sharp}(x, y) & \text{with probability } p \\ \text{arbitrary} & \text{otherwise} \end{cases}$$



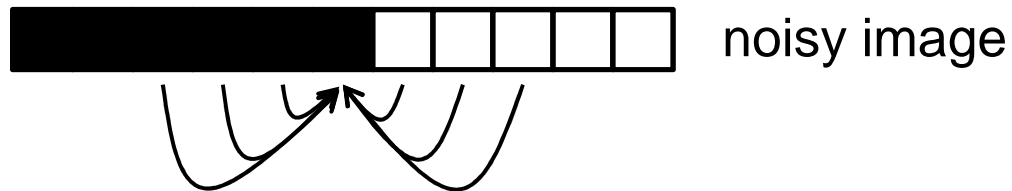


# Statistical Noise

- basic idea: averaging (smoothing)



- works well in homogeneous areas, but fails at grey level edges



# Smoothing Filters

- rectangular filter

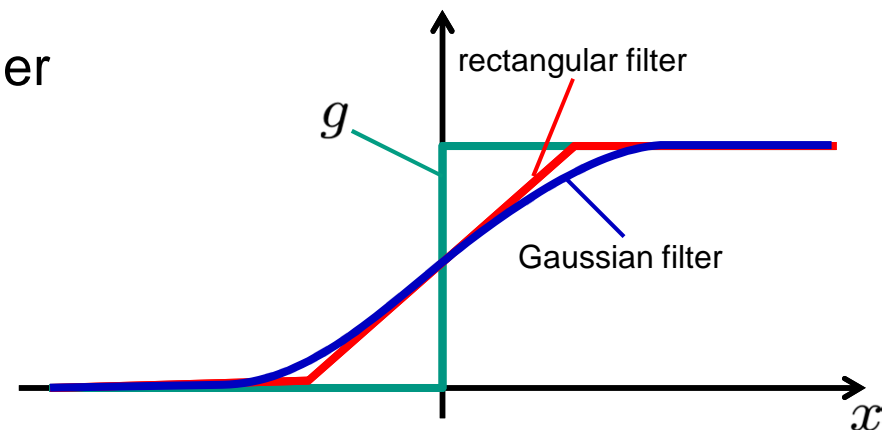
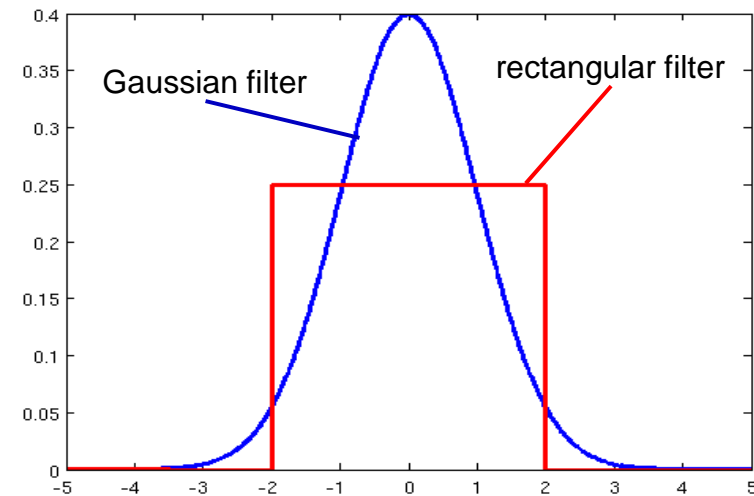
$$f(x) = \begin{cases} \frac{1}{a} & \text{if } |x| < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

the larger parameter  $a$ , the stronger smoothing

- Gaussian filter

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$$

the larger parameter  $\sigma$ , the stronger smoothing

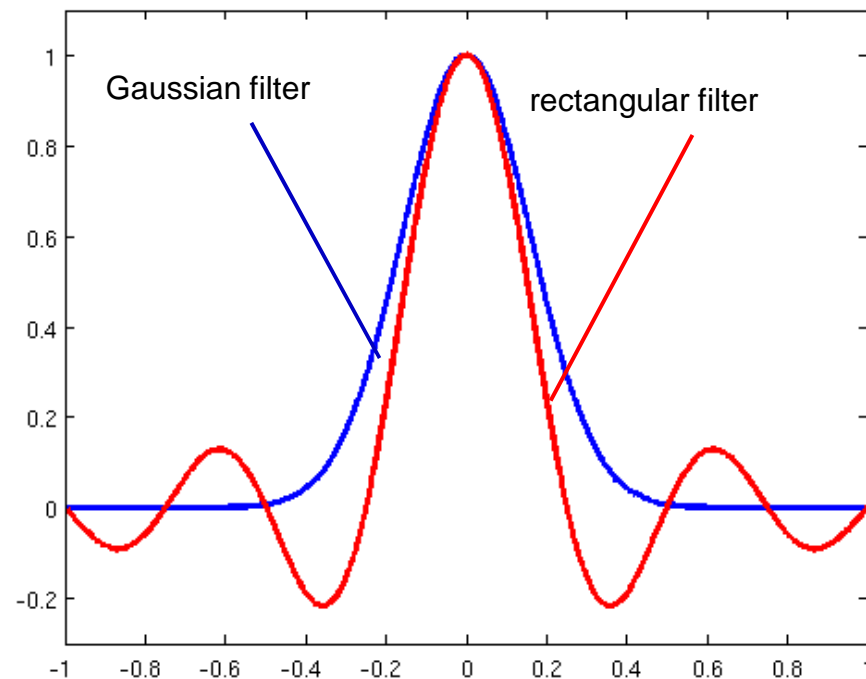


# Smoothing Filters

- Fourier transform of smoothing filters

$$\mathcal{F}(\text{rect.filter})(k) = \text{sinc}(ak) = \begin{cases} \frac{\sin(\pi x)}{\pi x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

$$\mathcal{F}(\text{Gauss.filter})(k) = e^{-2\pi^2\sigma^2k^2}$$



→ Smoothing: low pass filtering

# Smoothing Filters for Images

– rectangular filter masks:

$$\frac{1}{9} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

3x3 square mask

$$\frac{1}{25} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

5x5 square mask

$$\frac{1}{19.8} \cdot \begin{bmatrix} 0.1 & 0.8 & 1 & 0.8 & 0.1 \\ 0.8 & 1 & 1 & 1 & 0.8 \\ 1 & 1 & 1 & 1 & 1 \\ 0.8 & 1 & 1 & 1 & 0.8 \\ 0.1 & 0.8 & 1 & 0.8 & 0.1 \end{bmatrix}$$

5x5 disc mask

# Discrete Convolution cont.

– Gaussian filter masks:

$$\frac{1}{39} \cdot \begin{bmatrix} 1 & 4 & 1 \\ 4 & 19 & 4 \\ 1 & 4 & 1 \end{bmatrix}$$

3x3 mask,  $\sigma^2=0.25$

$$\frac{1}{273} \cdot \begin{bmatrix} 1 & 4 & 7 & 4 & 1 \\ 4 & 16 & 26 & 16 & 4 \\ 7 & 26 & 41 & 26 & 7 \\ 4 & 16 & 26 & 16 & 4 \\ 1 & 4 & 7 & 4 & 1 \end{bmatrix}$$

5x5 mask,  $\sigma^2=1$

– binomial filter masks

approximations to Gaussian masks using binomial coefficients  $\binom{n}{k}$

$$\begin{matrix} \binom{2}{0} \cdot \binom{2}{0} & & \binom{2}{1} \cdot \binom{2}{0} \\ & \nearrow & \nearrow \\ \binom{2}{0} \cdot \binom{2}{1} & & \binom{2}{1} \cdot \binom{2}{1} \\ & \nwarrow & \nwarrow \\ \binom{2}{0} \cdot \binom{2}{2} & & \binom{2}{1} \cdot \binom{2}{1} \end{matrix}$$

$$\frac{1}{16} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

3x3 mask

$$\frac{1}{256} \cdot \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

5x5 mask

## Smoothing Filters cont.

Gaussian, sigma = 7



# Bilateral filter

## Gaussian filter

- filter mask independent of image content

$$\underbrace{\tilde{g}(u, v)}_{\text{gray level after filtering}} \propto \sum_{i,j} \underbrace{\left( e^{-\frac{1}{2} \frac{i^2+j^2}{\sigma^2}} \right)}_{\text{distance dependent weight}} \cdot \underbrace{g(u+i, v+j)}_{\text{gray level}}$$

- smooth over edges and gross outliers

## Bilateral filter

- filter mask dependent on image content

$$\underbrace{\tilde{g}(u, v)}_{\text{gray level after filtering}} \propto \sum_{i,j} \underbrace{\left( e^{-\frac{1}{2} \frac{\|g(u+i, v+j) - g(u, v)\|^2}{\rho^2}} \right)}_{\text{content dependent weight}} \cdot \underbrace{\left( e^{-\frac{1}{2} \frac{i^2+j^2}{\sigma^2}} \right)}_{\text{distance dependent weight}} \cdot \underbrace{g(u+i, v+j)}_{\text{gray level}}$$

- reduces smoothing at edges and gross outliers

## Bilateral Filters cont.

Gaussian filter

$$\sigma = 7$$



bilateral filter  
 $\sigma = 7, \rho = 20$





# Salt-and-Pepper Noise

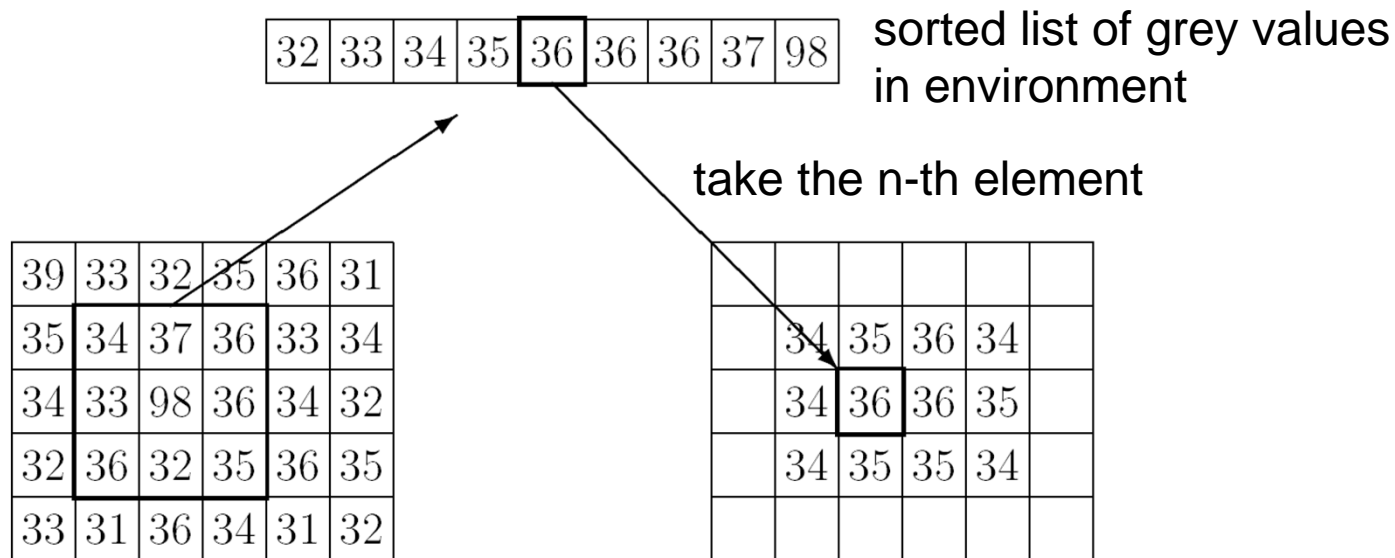


Gaussian  
filter



→ smoothing not appropriate for salt-and-pepper noise

# Median filter



## median filter:

- sort grey values in environment around reference pixel
- take the grey value in the middle of the sorted list

# Median filter



Gaussian filter



median filter



# SUMMARY: IMAGE PREPROCESSING

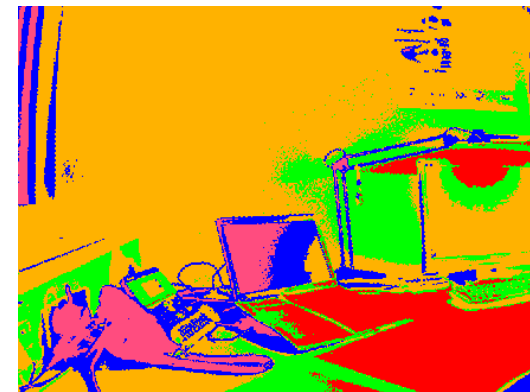
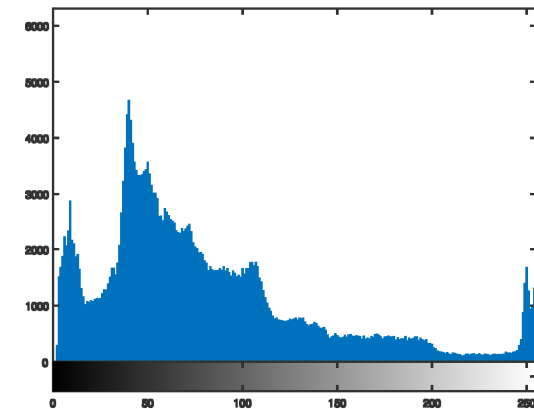
# Summary

- **sampling**
  - Moiré patterns
  - sampling theorem
  - Fourier transform
- **quantization**
- **blur and noise**



## Summary cont.

- **sampling**
- **quantization**
  - discrete grey values
  - histogram transformation
  - high dynamic range imaging
- **blur and noise**



## Summary cont.

- **sampling**
- **quantization**
- **blur and noise**
  - convolution
  - models of blur and noise
  - optimal image restoration (Wiener deconvolution)
  - smoothing filters
    - rectangular
    - Gaussian
    - bilateral
  - median filter

